所別：**數學系碩士班**　**不分組**科目：**線性代數**

(a) (10 分) Show that $\langle \cdot, \cdot \rangle$ is an inner product on V.

(b) (7 分) What is the adjoint operator T_A^* of T_A? Explain your answer.

(c) (8 分) Let $W = \{X \in V; \mathrm{Tr}(X) = 0\}$. Compute the orthogonal complement $W^\perp$ of W by giving an orthonomal basis for $W^\perp$. What is $\dim W^\perp$? Explain your answer.

5. (15 分) Let $L_i(x_1, x_2, \ldots, x_n) = \sum_{j=1}^{n} a_{i,j} x_j$, $a_{i,j} \in \mathbb{R}$ for $i = 1, \ldots, m$. Let

$$W = \{(b_1, \ldots, b_n) \in \mathbb{R}^n; L_i(b_1, \ldots, b_n) = 0, \ i=1,2,\ldots,m\}$$

be the subspace of $\mathbb{R}^n$ determined by the common zeros of the linear functionals $L_1, L_2, \ldots, L_m$. Let $f(x_1, x_2, \ldots, x_n)$ be a linear functional such that $f(b_1, \ldots, b_n) = 0$ for all $(b_1, \ldots, b_n) \in W$. Prove or disprove that there exist $\lambda_1, \lambda_2, \ldots, \lambda_m \in \mathbb{R}$ such that $f = \sum_{i=1}^{m} \lambda_i L_i$