

所別：數學系碩士班 不分組 科目：機率與統計

每題二十分

1. A manufacturing process produces an average of 10% defective items. A customer purchases 10 items. What is the probability that two or more of the items purchased are defective?
2. Three fair coins are tossed. Let  $Y$  be the number of heads and  $Z$  the number of tails. Then for each of the following random variables, find  $P\{X \leq x\}$ .
  - (a)  $X = Y - Z$ .
  - (b)  $X = \max\{Y, Z\}$ .
3. (a) Prove that when  $X$  has the binomial distribution, then
$$P\{X \text{ is even}\} = \frac{1}{2}\{1 + (q - p)^n\}.$$
(b) Prove that when  $X$  has the Poisson distribution,  $P\{X \text{ is even}\} = \frac{1}{2}\{1 + e^{-2\lambda}\}$ .
4. Let  $X$  and  $Y$  be independent identically distributed random variables, both uniformly distributed on  $(0, 1)$ . Find the probability density functions of the following.
  - (a)  $X + Y$
  - (b)  $X - Y$
5. Let  $f_{X,Y}(x, y) = (\sqrt{2}e^{-x})(\sqrt{2}e^{-y})$ ,  $0 < x < y < \infty$ . Show that  $X$  and  $Y$  are not independent random variables.

參考用