台灣聯合大學系統九十二學年度轉學生入學試題卷

類組：B-5　　年級：3 節次：1 科目：高等微積分　　

1. (20%) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)(x-1)^n.$$

Give sufficient reasons to support your answer.

2. For $\alpha > 0$, we denote by H_α the set of all functions on the bounded closed interval $[a, b]$ such that

$$|f(x) - f(y)| \leq M|x-y|^\alpha$$

for some constant M and all points $x, y \in [a, b]$.
(a) (10%) Show that if $\alpha < \beta$, then $H_\beta \subseteq H_\alpha$.
(b) (10%) Show that if $\alpha > 1$, then H_α contains the constant functions only.

3. (15%) A function $f(x)$ on the real line is called periodic if there is a constant $p > 0$ such that $f(x+p) = f(x)$ for all real x. Show that a continuous periodic function on the real line is uniformly continuous there.

4. (20%) Let $\{f_n\}$ be a sequence of continuous functions on $[a, b]$ converging uniformly to a function f. Let $\{x_n\}$ be a sequence of points in $[a, b]$ converging to a point c. Show that

$$\lim_{n\to\infty} f_n(x_n) = f(c).$$

5. Let

$$P(x,y) = -\frac{y^3}{(x^2+y^2)^2}, \qquad Q(x,y) = \frac{xy^2}{(x^2+y^2)^2}$$

and let Ω be a plane region with the origin in its interior and with a simple closed curve C_1 as its boundary. Let C_2 be another simple closed curve lying in the interior of Ω and enclosing the origin.
(a) (15%) Prove that the line integrals

$$\oint_{C_1} P\,dx + Q\,dy = \oint_{C_2} P\,dx + Q\,dy.$$

(b) (10%) Let C_3 be the limaçon $r = 2 + \cos\theta$ (in polar coordinates). Evaluate the line integral

$$\oint_{C_3} P\,dx + Q\,dy.$$